Multi-objective Optimization of PV-SOFC-GT-Electrolyser Hybrid System

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ABSTRACT

This study shows the design of a new hybrid power generation system, photovoltaic panel (PV)-coupled solid oxide fuel cell (SOFC) and gas turbine (GT)-electrolyser. Three objectives (cost, pollutant emissions, and reliability), which are usually in conflict, are considered simultaneously. The design of a hybrid system, considering the three mentioned objectives, poses a very complex problem of optimization. A multi-objective optimization method (PESA) is considered to obtain the best combinations for the hybrid system. In this work, the effect of panel’s angle change and SOFC-GT fuel type are considered too. In order to study the effect of fuel price, this study is done about two fuel prices: Iran fuel price and international fuel price.

INTRODUCTION

Nowadays, satisfying the energy demand is a sensitive global topic. In parallel, it is also necessary to limit fossil fuels consumption for both sustainability and energy self-sufficiency issues. Therefore, in the global warming context, the development of intermittent renewable energy like photovoltaic (PV) could partly respond to the long term prospects of an increasing energy demand. The conversion of solar is clean, silent and reliable, with reduced maintenance costs, and with small ecological impact. Moreover, sunlight is for free, practically inexhaustible and involves no polluting residues or greenhouse gases emission. Along with these advantages, the electric power production systems using as primary sources exclusively the solar, pose technical problems due to day–night–summer–winter alternation. Such systems would be especially adapted when coupled with a storage system and an auxiliary system that would both increase the productivity of power plants and reduce the areas dedicated to energy production. Such systems, dedicated to decentralized production of electric power based on combined sources of energy, are called hybrid systems. Baniasad Askari and Ameri [1] studied a simple optimization method for calculating the optimum configurations of photovoltaic–battery (PV–bat) hybrid system with high reliability and minimum cost. The proposed method was applied to design a PV–bat system to supply a typical load requirement in a remote region in Kerman, Iran. Nafeh [2] sized a PV/Diesel generator hybrid energy system to meet the load for about 100% availability. The operation of the diesel generator and the number of the PV modules and batteries were optimized for a given load characteristic and a given diesel generator that would achieve a minimum initial cost and a desired depth of discharge for battery storage. Dufo and Bernal [3] optimized a PV-Diesel system by HOGA program (Hybrid Optimization by Genetic Algorithms) and compared it with a stand-alone PV-only system. Results show the economical advantages of the PV-hybrid system. Morera and Tudorache [4] designed an optimal integrated hybrid system for autonomous electric power production, based on the concurrent operation of a wind turbine and a Photovoltaic (PV) system, backed up by a diesel generator. The optimization of the proposed hybrid system is based on the logistic type numerical models implemented in the HOMER software package. Rehman and Al-Hadhrami [5] studied a PV-diesel hybrid power system with battery backup for a village being fed by diesel generated electricity to replace part of the diesel by solar. Four generators of different rated powers, diesel prices of 0.2–1.2US$/lit, different sizes of batteries and converters were used to find an optimal power system for the village. Khatib et al [6] performed an optimization to build an integrated hybrid PV/diesel generator system for zero load rejection for Malaysia.
The optimization presented in their work aims to calculate the optimum capacities of a PV array and diesel generator, which investigates the minimum system cost. Yamegueu et al. [7] studied the results of an experimental study of a PV/diesel hybrid system without storage. Their experimental results showed that the sizing of a PV/diesel hybrid system by taking into account the solar radiation and the load-demand profile of a typical area may lead the diesel generator to operate near its optimal point. Also, for a reliability of a PV/diesel hybrid system, the rated power of the diesel generator should be equal to the peak load. Baniasad and Ameri [8] used PV-diesel-battery power systems to meet typical load requirements in a remote region in Kerman, Iran. They used a simple optimization method to determine systems with high reliability and low cost. The combination of solar panels with fuel cells and electrolyser is often studied, but usually Proton exchange membrane (PEM) fuel cells are used. Nbil et al. [9] presented a hybrid energy system combining variable speed wind turbine, solar photovoltaic, electrolyser, and PEM fuel cell generation systems to supply continuous power to residential power applications as stand-alone loads. Silva et al. [10] presented an economical assessment and optimization of a hybrid distributed generation system, comprised of a PV system, PEM fuel cell, and batteries as a potential source of energy for isolated communities in the Amazon region. Erdogan et al. [11] proposed a photovoltaic/wind/electrolyser/PEM fuel cell hybrid power system for stand-alone applications demonstrated by a mobile house. They showed that different renewable sources can be used simultaneously to power off-grid applications. Lee et al. [12] described the Power managements of a UAV's hybrid electric propulsion systems. They considered three electric propulsion systems with different power sources, i.e. solar cells, fuel cells, and batteries. Each power source was modelled in Matlab/Simulink and integrated into the power system. Fuel cells and batteries simulation processes were verified via a comparison between the simulation results and available flight test results of UAVs. Sadeghi and Ameri [13] compared the combination of different auxiliary systems with solar panels and batteries in terms of economical efficiency, ecological compatibility, and reliability. Auxiliary systems include diesel generator, gas generator, solid oxide fuel cell, and micro gas turbine. They also considered the effect of fuel price, technology development in SOFC cost, changes in the power of auxiliary power system and also change in the maximum number of panels. Because of high temperature products, SOFCs are suitable to combine with gas turbine or micro gas turbine. Cheddie [14] indirectly coupled the SOFC system to the GT in order to minimize the disruption to the GT operation. Santin et al. [15] considered a study of SOFC–GT hybrids for operation with liquid fuels. Two liquid fuels were investigated, methanol and kerosene, in four layouts, taking into account different fuel processing strategies. Because of the large number of variables usually considered and the mathematical models applied, classical optimization techniques may consume excessive CPU time or even prove unable to take into account all the characteristics associated with the posed problem. During the last 3 decades, heuristic techniques have been applied. One of the most used heuristic techniques has been the multi-objective evolutionary algorithms (MOEAs). Dufo and Bernal [16] applied the strength Pareto evolutionary algorithm to the multi-objective design of isolated hybrid systems, minimizing both the total cost and the unmet load. They also studied a triple multi-objective design of isolated PV–wind–diesel–hydrogen–minimized battery system and simultaneously, the total cost throughout the useful life of the installation, pollutant emissions (CO2) and unmet load by a multi-objective evolutionary algorithm (MOEA) and a genetic algorithm (GA) [17]. Sadeghi and Ameri [18] presented a multi-objective optimization method for calculating the optimum configurations of photovoltaic–battery systems with high reliability and minimum cost for different tilt angles of the panels. Iran holds the world's second-largest proven natural gas reserves, surpassed only by Russia in the world, and also holds the world's fourth-largest proved crude oil reserves [19]. Also, there is a natural gas pipeline in most parts of the country. On the other hand, Iran has a good solar energy potential. Therefore, using solar radiation, natural gas and petroleum products for local power generation are recommended. In this study, the combination of photovoltaic panels (PV), coupled solid oxide fuel cell (SOFC) and gas turbine (GT), and electrolyser are considered for power generation. The number of PV panels, the maximum number of PV panels, the power of SOFC-GT, and the power of electrolyser can be changed. A multi-objective optimization method (PESA) is considered to obtain the best hybrid systems from economical, ecological, and reliability point of view. Also in this work, the effect of panel's angle changes and SOFC fuel type are considered. This study is done about two fuel prices: Iran fuel price and international fuel price.

2. SYSTEM DESCRIPTION

Figure 1. illustrates a schematic diagram of the PV-coupled SOFC and GT-electrolyser hybrid system. The proposed system consists of the PV panels, a coupled SOFC-GT system, an electrolyser, the hydrogen tanks, the inverters, and other related components. The load demand is determined for each hour. If the generated power from PV panels is greater than the load demand, the excess power converts to hydrogen by electrolyser as much as the power of electrolyser and
stores in hydrogen tanks. If the generated power from PV panels is lower than the load demand, the remaining power is generated by SOFC-GT with hydrogen as fuel. Finally, if the stored hydrogen is not enough to generate the remaining power, SOFC-GT uses other fuels (methane, propane, diesel fuel or ethanol) to generate power. However, if the remaining power is greater than SOFC-GT power, this hour marks an unmet load. The SOFC and PV panel generated power is DC current, which is converted into the AC current by inverters.

Figure 1. The hybrid power generation system.

Figure 2. The coupled SOFC-Gas Turbine system with Steam Reforming.

Figure 3. Monthly average hourly load.

Figure 4. Monthly average daily radiation.

3. LOAD DEMAND

In the present work, load demand is a representation of remote households in Kerman, which are far from the utility grid. The measured annual average electric energy consumption of 500 typical households is considered. The diagram of sample load has been plotted in Figure 3, which shows mean electrical load for every month.

4. MATHEMATICAL MODELING OF COMPONENT

4.1. PV panels

The solar energy calculations are made by using the hourly solar radiation data. The electricity generated by the PV systems is directly related to the solar energy received by the PV panels while the PV panels can be placed at different tilt angles and orientations. Most local solar observatories only provide solar irradiance data on a horizontal plane (Radiation in Kerman for different month of the year has been shown in Fig 4.) Thus, an estimate of the total solar radiation incident on any required sloping surfaces is needed. The HDKR model (Hay, Davies, Klucher, Reindl model) [20] is utilized to estimate the total solar radiation on the tilted surface:
\[ I_1 = (I_o + I_A)R_b + I_o (1 - A_t) \left( \frac{I_o + I_A}{2} \right) \left( 1 + \sin^2 \left( \frac{\theta}{2} \right) \right) + \left( I_o + I_A \right) R_p \left( \frac{I_o + I_A}{2} \right) \]

where \( I_o \) and \( I_A \) are direct normal and diffuse solar radiations, \( A_t \) is the anisotropy index, and \( R_b \) is the geometric factor, which are defined as below:

\[ A_t = \frac{b}{l_o} \]  
\[ R_b = \frac{\cos \theta}{\cos \theta_p} \]  

In the above relations, \( I_o \) is the horizontal extraterrestrial radiation on a horizontal surface; \( \theta \) and \( \theta_p \) are incidence and zenith angles, respectively. \( f \) is the cloudiness factor and is given by the following equation:

\[ f = \frac{b}{b + I_d} \]  

In equation (1), \( \beta \) is the slope of PV panels and \( \rho_g \) the ground reflectance (also called Albedo), is the fraction of solar radiation incident on the ground that is reflected. A typical value of ground reflectance for grass-covered areas is 20 per cent, snow-covered area is 70 per cent, grass-plot area is 30 per cent, and desert dry lands are 45 per cent. In this article, the ground reflectance value is considered to be 45 per cent according to the Kerman climate (dry/desert-covered area).

Hourly power output from PV system is given by:

\[ P_{PV} = I_m \eta_{lm} \eta_{pc} P_t A_{PV} \]  

where \( A_{PV} \) is the total area of the PV modules in \( m^2 \), \( \eta_{lm} \) is the module reference efficiency (0.11), \( P_t \) is the packing factor (0.91), and \( \eta_{pc} \) is the power conversion efficiency (0.83). The module reference efficiency \( \eta_{lm} \) can be estimated from the current and voltage of the PV module at maximum power point:

\[ \eta_{lm} = \frac{C_{mp} V_{mp}}{G A_{cs}} \]  

where \( C_{mp} \) is the current at maximum power point \( A \), \( V_{mp} \) is the voltage at maximum power point \( V \), and \( A_{cs} \) is the area of a single PV module \( m^2 \). The solar radiation at reference condition \( G \) in equation (6) is 1000W/m².

4.2. Auxiliary system

The SOFC is a complete solid-state device and uses an oxide ion-conducting ceramic material as the electrolyte. It is therefore more simple in concept than all the other fuel cell systems, as only two phases (gas and solid) are required. The high operating temperatures mean that precious metal electrocatalysts are not needed. Both hydrogen and carbon monoxide can act as fuels in the SOFC. A negatively charged ion (O⁻) is transferred from the cathode through the electrolyte to the anode. Thus, product water is formed at the anode. Until recently, SOFCs have all been based on an electrolyte of zirconia stabilized with the addition of a small percentage of yttria. Above a temperature of about 800°C, zirconia becomes a conductor of oxygen ions (O²⁻), and typically the state-of-the-art zirconia-based SOFC operates between 900 and 1000°C. This is the highest operating temperature of all fuel cells, which presents both challenges for the construction and durability, and also opportunities, for example, in combined cycle applications. A SOFC generates electrical power by continuously converting chemical energy of a fuel into electrical energy through an electrochemical reaction. The fuel cell itself has no moving parts, making it quiet and reliable. The system has higher efficiency compared to conventional combustion engines, because it is not limited by Carnot efficiencies. The power output of the SOFC is given by [21, 22]:

\[ \dot{W}_{SOFC} = N_{cell} V_{fc} I_{fc} \]  

where, \( N_{cell} \) is the number of single fuel cell in the stack, \( V_{fc} \) is the cell voltage and the stack current is given by:

\[ I_{fc} = I_{c} A_{fc} \]  

where \( I_{c} \) is the current density and \( A_{fc} \) is the effective cross sectional area.

The gas turbine is an internal combustion engine that uses combustion products as the working fluid. The engine extracts chemical energy from fuel and converts it to mechanical energy. High temperature high-pressure gas enters a turbine, where it expands down to the exhaust pressure, producing a shaft work output in the process. The turbine shaft work is used to drive the compressor and the electric generator that is coupled to the shaft. It is assumed that the cycle reaches a steady state; the pressure drops in the connection tubes are neglected; there is no heat transfer with the environment; and the gas turbine and compressors have a given isentropic efficiency, respectively. The SOFC outlet gas enters post-combustor. It is assumed that the combustion process is performed adiabatically and completely. The isentropic efficiency of the turbine [23],

\[ \eta_{GT} = \frac{h_{in} - h_{out}}{h_{in} - h_{out,s}} \]  

The generating power can be given as:

\[ \dot{W}_{GT} = \dot{m} (h_{in} - h_{out}) \]  

The isentropic efficiency of compressors or pump is:

\[ \eta_{com or pump} = \frac{h_{out,s} - h_{in}}{h_{out} - h_{in}} \]  

The actual compression power is:

\[ \dot{W}_{pump} = \dot{m} (P_{out} - P_{in}) \]  

Table 1. shows the Operating parameters of coupled SOFC-Gas Turbine system.
The specific fuel consumption is defined as the fuel consumption required to produce 1 kWh of energy, and it is equal to the hourly fuel consumption for supplying a given load during 1 h. According to Skarstein and Uhlen [24], the hourly fuel consumption can be approximated as follows:

$$F.C. = A \times P(t) + B \times P_n$$  \hspace{1cm} (14)

Where A and B are constants that are different for various fuels, P(t) is the power generated at t moment, and P_n is the rated/nominal power. The rate of SOFC hydrogen usage in moles per second is:

$$H_2_{usage} = \frac{W_{total}}{2N_FU_r}$$  \hspace{1cm} (15)

where F is the Faraday constant and U_r = 0.85 is the fuel utilization. The rate of SOFC fuel consumption in moles per second is given as:

$$Fuel\, Consumption = \frac{W_{total}}{2N_FU_r}$$  \hspace{1cm} (16)

where n is the sum of the number of H_2 and CO that yield from reforming of one mole of fuel. The rate of SOFC-GT fuel consumption in moles per second is given as:

$$Fuel\, Consumption = \frac{W_{total}}{2N_FU_r}$$  \hspace{1cm} (17)

where r is the ratio of SOFC-GT power to SOFC power that depends on the type of fuel and the type of SOFC reforming. Table (2) shows the constant of A that is obtained from equation (30) and the constant of r.

**TABLE 1.** Operating parameters of coupled SOFC-Gas Turbine system.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ambient temperature (K)</td>
<td>298</td>
</tr>
<tr>
<td>Ambient pressure (bar)</td>
<td>1.013</td>
</tr>
<tr>
<td>Fuel compressor isentropic efficiency</td>
<td>0.85</td>
</tr>
<tr>
<td>Air compressor isentropic efficiency</td>
<td>0.85</td>
</tr>
<tr>
<td>Turbine isentropic efficiency</td>
<td>0.87</td>
</tr>
<tr>
<td>Inverter efficiency</td>
<td>0.92</td>
</tr>
<tr>
<td>SOFC operating pressure (bar)</td>
<td>3.432</td>
</tr>
<tr>
<td>SOFC operating temperature (K)</td>
<td>1173</td>
</tr>
<tr>
<td>SOFC fuel utilization factor</td>
<td>0.85</td>
</tr>
<tr>
<td>Cell operating voltage (v)</td>
<td>0.6</td>
</tr>
<tr>
<td>SOFC effective cross sectional area (m²)</td>
<td>0.05</td>
</tr>
<tr>
<td>SOFC operating current density (A/m²)</td>
<td>3330</td>
</tr>
</tbody>
</table>

**4.3. Electrolyser** Electrolysis is an electrochemical process in which electrical energy is the driving force of chemical reactions. The electrolysis of water is considered a well-known principle to produce oxygen and hydrogen gas. The core of an electrolysis unit is an electrochemical cell, which is filled with pure water and has two electrodes connected with an external power supply.

At a certain voltage, which is called critical voltage, between both electrodes, the electrodes start to produce hydrogen gas at the negatively biased electrode and oxygen gas at the positively biased electrode. The amount of gases produced per unit time is directly related to the current that passes through the electrochemical cell. Typical cell voltages are 1.80 to 2.05 V [25]. Electricity requirement is about 4.6 kWh/Nm³H₂.

**4.4. Inverter** A power inverter, or inverter, is an electrical device that changes direct current (DC) to alternating current (AC); the converted AC can be at any required voltage and frequency with the use of appropriate transformers, switching, and control circuits. In this study, ten 10KW inverters with 92% efficiency have been used.

**5. OBJECTIVE FUNCTIONS**

The objective functions are:

- The CO₂ emission: (kg/year).
- The annualized cost: ANC ($/year).

The loss of power supply percent: LPSP (%).

**5.1. CO₂ Emission** Table 3 shows CO2 emission for different fuels. The hours of auxiliary systems operation and therefore, fuel consumptions are specified, so emission of different auxiliary systems will be determined.

**5.2. Annualized cost** In finance, the annualized cost is the cost per year of owning and operating an asset over its entire lifespan. ANC is often used as a decision making tool when comparing investment projects of unequal lifespans. In the present study, to compare different configurations of economical aspects, annualized cost is used. In order to calculate ANC, annualized initial capital cost, annualized replacement cost, and annualized operating and maintenance cost will be added.

**Annualized initial capital cost:**

$$C_{acap} = C_{cap} \times CRF(i, R)_{proj}$$  \hspace{1cm} (18)

$C_{acap}$, $C_{cap}$, CRF, i, $R_{proj}$ are annualized initial capital cost, initial capital cost, capital recovery factor, real interest rate, and system lifespan respectively.

**Real interest rate:**

$$i = \frac{f - i'}{f + 1}$$  \hspace{1cm} (19)

i’ and f are nominal interest rate and inflation, respectively.

**Capital recovery factor:**

$$CFR(i, R_{proj}) = \frac{(1+i)^{R_{proj}} - 1}{(1+i)^{R_{proj}} - 1}$$  \hspace{1cm} (20)

**Annualized replacement cost:**

$$C_{arep} = C_{rep} \times SFF(i, R_{comp}) \times S \times SFF(i, R_{proj})$$  \hspace{1cm} (21)

$C_{arep}$, $C_{rep}$, $SFF$, $R_{comp}$, S are annualized replacement cost, replacement cost, ratio of capital recovery factor,
sinking fund factor, and lifespan of component and salvage value, respectively. Sinking fund factor: 

$$SFF(i, N) = \frac{i}{(1+i)^N - 1}$$  \hspace{1cm} N = R_{comp}R_{proj} \hspace{1cm} (22)$$

Salvage value: 

$$S = C_{rep} \times R_{rem} \hspace{1cm} R_{rem} = R_{comp} - (R_{proj} - R_{rep}) \hspace{1cm} (23)$$

Ratio of capital recovery factor: 

$$f_{rep} = \begin{cases} \frac{CRF(i, R_{proj})}{CRF(i, R_{rep})} & \text{if } R_{rep} > 0 \\ 0 & \text{if } R_{rep} = 0 \end{cases} \hspace{1cm} (24)$$

Operating and maintenance costs are usually annualized. 

Annualized cost: 

$$ANC = C_{acap} + C_{arep} + C_{a \text{ O&M}} \hspace{1cm} (25)$$

Fuel prices in Iran and other places are different. In the present study, the comparison is done based on two fuel prices:

1. Iran fuel price
2. International fuel price

Table 4 shows the fuel prices. Table (5) shows the initial, replacement, operation and maintenance cost of different components. This table also shows life span and power of the components.

### TABLE 2. constants for different fuels

<table>
<thead>
<tr>
<th></th>
<th>Hydrogen</th>
<th>Methane</th>
<th>Propane</th>
<th>Ethanol</th>
<th>Diesel fuel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant of A</td>
<td>0.558</td>
<td>0.155</td>
<td>0.213</td>
<td>0.227</td>
<td>0.148</td>
</tr>
<tr>
<td>(m$^3$.KW/h)</td>
<td>(m$^3$.KW/h)</td>
<td>(lit.KW/h)</td>
<td>(lit.KW/h)</td>
<td>(lit.KW/h)</td>
<td></td>
</tr>
<tr>
<td>Constant of r</td>
<td>1.6112</td>
<td>1.4528</td>
<td>1.4789</td>
<td>1.5716</td>
<td>1.5078</td>
</tr>
<tr>
<td>W$_{act}$(KW)/500 KW Hybrid system</td>
<td>189.67</td>
<td>155.86</td>
<td>161.91</td>
<td>181.85</td>
<td>168.39</td>
</tr>
</tbody>
</table>

### TABLE 3. CO2 emission per unit fuel consumption for different fuels

<table>
<thead>
<tr>
<th></th>
<th>CO2 emission</th>
</tr>
</thead>
<tbody>
<tr>
<td>Methane</td>
<td>1.782 (kg/m$^3$ Methane)</td>
</tr>
<tr>
<td>Propane</td>
<td>1.53 (kg/lit Propane)</td>
</tr>
<tr>
<td>Ethanol</td>
<td>1.497 (kg/lit Ethanol)</td>
</tr>
<tr>
<td>Diesel Fuel</td>
<td>2.329 (kg/lit Diesel fuel)</td>
</tr>
</tbody>
</table>

### TABLE 4. Different fuel prices

<table>
<thead>
<tr>
<th></th>
<th>International fuel price [26]</th>
<th>Iran fuel price [27, 28]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Methane</td>
<td>0.123($/m^3$)</td>
<td>0.084($/m^3$)</td>
</tr>
<tr>
<td>Propane</td>
<td>0.232($/lit$)</td>
<td>0.04($/lit$)</td>
</tr>
<tr>
<td>Ethanol</td>
<td>0.667($/lit$)</td>
<td>0.74($/lit$)</td>
</tr>
<tr>
<td>Diesel Fuel</td>
<td>0.852($/lit$)</td>
<td>0.14($/lit$)</td>
</tr>
</tbody>
</table>

### TABLE 5. Specifications of different components [29, 30]

<table>
<thead>
<tr>
<th></th>
<th>Power</th>
<th>Initial capital cost ($/KW)</th>
<th>Replacement cost ($/KW)</th>
<th>O&amp;M cost per year ($/KW)</th>
<th>Life span (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV panel</td>
<td>200 W</td>
<td>1000-3000</td>
<td>0</td>
<td>0.0025</td>
<td>25</td>
</tr>
<tr>
<td>Electrolyser</td>
<td>-</td>
<td>450</td>
<td>405</td>
<td>0.1</td>
<td>15</td>
</tr>
<tr>
<td>Inverter</td>
<td>10×10 KW</td>
<td>200-400</td>
<td>180-360</td>
<td>0.0015</td>
<td>15</td>
</tr>
<tr>
<td>SOFC-GT</td>
<td>-</td>
<td>1100</td>
<td>990</td>
<td>0.0086</td>
<td>15</td>
</tr>
</tbody>
</table>

**5.3. Loss of power supply percent**

Reliability of the system is expressed in terms of loss of power supply percent (LPSP). The objective function, LPSP, can be described by:

$$LPSP = \frac{\sum_{n=1}^{N_h} \text{hours} \times (P_{\text{supply}}(t) - P_{\text{need}}(t))}{N_h} \times 100 \hspace{1cm} (26)$$

In the above relation, $N_h$ is the number of time intervals (8760, number of hours in a year).

6. **MULTI-OBJECTIVE OPTIMIZATION EVOLUTIONARY ALGORITHM**

In this section, the multi-objective design problem is mathematically formulated and the basic concepts used by the MOEAs are defined. Finally, the applied MOEA (PESA) is also described, which searches the best combination of components minimizing ANC, and pollutant emissions. A multi-objective optimization problem can be defined as follows:

Minimize or maximize the objective functions included in the vector: 

$$f(x) = \{f_1(x), f_2(x), \ldots, f_k(x)\}$$

Satisfying the m restrictions of inequality and the p restrictions of equality:
3.3.1. Select two parents from EP with probability (1-\(P_c\)).
3.3.2. Recombine these two parents for obtaining one offspring
3.3.3. Mutate the offspring
3.3.4. Select one parent from IP with probability (1-\(P_c\)).
3.3.5. Mutate the parent to produce one offspring
3.3.6. Add the two obtained offspring into IP
4. Return to 3

This algorithm is in charge of finding the designs that manage to, simultaneously, minimize the ANC of the system, and the pollutant emissions. It has been developed using the MATLAB programming language. The algorithm (MOEA) can search for the configuration of PV panels, auxiliary system, and inverter which minimizes three objectives mentioned.

7. RESULTS

The energy from sunlight reaching the earth is a free and huge source of energy that can be exploited and used for generating electricity by photovoltaic panels. Given the discontinuous nature of solar energy, the use of a storage device to store energy during the day and supply it at night is essential. The storage device in this study is an electrolyser and H\(_2\) tanks. Moreover, using an auxiliary system is necessary to increase the reliability of the power generation system and cloudy days. One of the most efficient auxiliary systems is coupled SOFC-GT. This work, in addition to introducing this new hybrid system, explains the effect of different conditions on it. The most desirable hybrid system is the low cost and the least emission one. But cost and emission are in conflict with each other. In order to obtain optimum solutions (Solutions are different in the number of PV panels and the power of electrolyser), a multi-objective optimization (PESA) is used. Because of the SOFC operating conditions, various fuels can be used. Four specific fuels are considered to determine the best fuel from ecological and economical point of view. Diesel fuel: that is a fossil fuel which is portable and storable in liquid fuel tanks, and a conventional liquid fuel. Ethanol: that is a renewable fuel made from corn and other plant materials. Propane: a fossil fuel which is portable and storable in pressure vessels. Methane: a gas fuel which is transportable by pipelines.

Each photovoltaic panel occupies about 1.5 m\(^2\), so the maximum number of panels (MNP) is limited. The effect of this limitation has been considered in this study. Figure 5 shows the Pareto frontiers of four fuels for two different situations, when the maximum number of panels is limited to 1000 and when it is limited to 3000. This figure is for international fuel price, and the pollutant emissions. It has been developed using the MATLAB programming language. The algorithm (MOEA) can search for the configuration of PV panels, auxiliary system, and inverter which minimizes three objectives mentioned.

The implemented multi-objective algorithm is based on PESA that is presented by Corne et al. [31]. It has approximately fast convergence, probably due to its higher elitism intensity and it also has good accuracy. PESA has two parameters concerning population size i.e. PI (the size of the internal population IP) and PE (the maximum size of the archive or external population). It has one parameter concerning the hyper-grid crowding strategy. The main steps in this algorithm are (i) Generate and evaluate each of an initial internal population (IP) of PI chromosomes and initialize the external population (EP) to the empty set.(ii) Incorporate the non-dominated members of IP into EP.(iii) If a termination criterion has reached then stop, returning the set of chromosomes in EP as the result.

Otherwise, delete the current contents of IP and repeat the following until PI new candidate solutions have been generated. With probability \(P_c\), select two parameters from EP. Produce a single child via uniform crossover and mutate the child via bit-flip mutation. With probability (1-\(P_c\)) select one parent and mutate it to produce a child. (iv) Repetition of the same process.

1. Generate and evaluate each of an initial internal population (IP) of PI chromosomes.
2. Initialize the external population (EP) as empty set.
3. For \(t=1\) to Number of Generations
   3.1. Incorporate the non-dominated members of IP into EP.
   3.2. Delete the current content of IP.
   3.3. Until obtaining new solution of PI.
   3.3.1. Select two parents from EP with probability \(P_c\).
   3.3.2. Recombine these two parents for obtaining one offspring
   3.3.3. Mutate the offspring

\(g_i(x)  \geq 0\) \(i=1,2,\ldots,m\)
\(h_i(x)  = 0\) \(i=1,2,\ldots,p\)

Where \(x\) is a vector whose elements are the decisive variables of the problem.

Concepts related to Pareto optimality are regularly used in most MOEAs. Because of this, the concepts of Pareto dominance, Pareto optimality, Pareto optimal set, and Pareto front are defined.

Pareto dominance: a vector \(u = (u_1, u_2,\ldots, u_k)\) is said to dominate \(v = (v_1, v_2,\ldots, v_k)\) (denoted by \(u \leq v\)) if and only if \(u\) is partially less than \(v\), i.e., \(\forall i \in \{1,2,\ldots,k\}; u_i  \leq v_i \land \exists i \in \{1,2,\ldots,k\}; u_i < v_i\).

Pareto optimality: a solution \(x \in \Omega\) is said to be Pareto optimal with respect to \(\Omega\) if and only if there is no \(x \in \Omega\) for which \(v = F(x') = (f_1(x'), f_2(x'), \ldots, f_k(x'))\) dominates \(u = F(x) = (f_1(x), f_2(x), \ldots, f_k(x))\).

Pareto optimal set: for a given MOP \(F(x)\), the Pareto optimal set \((P^*)\) is defined as follows:
\[P^* = \{x \in \Omega \mid \exists x' \in \Omega : F(x') \leq F(x)\}\]

Pareto front: for a given MOP \(F(x)\) and Pareto optimal set \(P^*\), the Pareto front \((PF^*)\) is defined as follows:
\[PF^* = \{u = F(x) = (f_1(x), f_2(x), \ldots, f_k(x)) \in \mathbb{R}^k \mid x \in P^*\}\]

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system and so the emission increase when the number of panels is less than 1000. On the other hand, because of the high fuel price, the annualized cost increases. As a result, annualized cost and emission are both increase and therefore, optimization algorithm selects the hybrid system with 1000 PV panels only; however, when the number of panels is more than 1000, the effect of the panel cost is greater than the effect of fuel price so, the Pareto frontier contains more than one solution. The difference between Figure 6. and Figure 5. is that Figure 6. is obtained for Iran fuel prices. Diesel fuel price is lower in Iran so the Pareto frontier of diesel fuel is similar to that of methane and propane. Therefore, the number of optimal solutions depends on the fuel price and MNP. Table 6. shows some of the results with approximately the same number of PV panels for comparison. The MNP is considered 3000 for the rest of the results of this study. Electing an appropriate power for the auxiliary system is very important. If the power of the auxiliary system is low, LPSP of the system increases i.e. the reliability of the system decreases. And if the power of the auxiliary system is high, annualized cost and emission of the system increase without any effect on its reliability. Figure 7. shows the Pareto frontiers of methane for four different power of auxiliary system in international fuel price. It is obvious that when the power of the auxiliary system decreases, annualized cost and emission will decrease. But as it is seen in Table 7. LPSP will increase when the power of the auxiliary system decreases. Table 7. shows that the optimum power for the auxiliary system is 500KW. Figure 8. shows the Pareto frontiers of methane for four different power of the auxiliary system in Iran fuel price, and Figure 9. shows the Pareto frontiers of diesel fuel for four different power of the auxiliary system in international fuel price. From Figures 8. and 9. it can be concluded that the fuel type and the fuel price do not influence the election of the power of the auxiliary system. The power of the auxiliary system is considered 500KW for the rest results of this study. Figure 10. shows the Pareto frontiers for four different fuels in international fuel price. It is obvious that methane is the best and ethanol is the worst fuel from ecological and economical points of view. But ethanol is a renewable fuel made from corn and other plant materials. Plants convert CO₂ to O₂ in photosynthesis process. Therefore, the ethanol production process decreases CO₂ emission but calculation of this reduction is difficult and it is not considered in this study. Because of this and according to the depletion of fossil fuel reserves, the renewable sources of energy will be in more attention in the near future. According to Figure 10. for places without gas pipeline, the best fuel is propane. Figure 11. compares these four fuels in Iran fuel price. Due to the lower price of diesel fuel in Iran, the difference between the Pareto frontiers of diesel and propane in Figure 11. is less than that in Figure 10.
### Table 6. Some of the optimum results for 500KW hybrid system

<table>
<thead>
<tr>
<th>MNP</th>
<th>Fuel</th>
<th>ANC ($/year)</th>
<th>CO2 emission (kg/year)</th>
<th>Number of panel</th>
<th>Power of electrolyser (KW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>CH₄</td>
<td>106908</td>
<td>214848</td>
<td>994</td>
<td>110</td>
</tr>
<tr>
<td></td>
<td>C₂H₆</td>
<td>130049</td>
<td>255383</td>
<td>977</td>
<td>110</td>
</tr>
<tr>
<td></td>
<td>CH₃OH</td>
<td>209658</td>
<td>263259</td>
<td>1000</td>
<td>110</td>
</tr>
<tr>
<td></td>
<td>C₂H₅OH</td>
<td>190058</td>
<td>267187</td>
<td>999</td>
<td>110</td>
</tr>
</tbody>
</table>

#### International fuel prices

- 3000: CH₄ 186247, C₂H₆ 196033, C₂H₅OH 229284
- 1000: CH₄ 102311, C₂H₆ 98671, C₂H₅OH 107919
- 3000: CH₄ 185978, C₂H₆ 134474, C₂H₅OH 205056
- 1000: CH₄ 98671, C₂H₆ 234100, C₂H₅OH 188460

#### Iran fuel prices

- 3000: CH₄ 106908, C₂H₆ 130049, C₂H₅OH 190058
- 1000: CH₄ 186247, C₂H₆ 196033, C₂H₅OH 229284
- 3000: CH₄ 185978, C₂H₆ 134474, C₂H₅OH 205056
- 1000: CH₄ 98671, C₂H₆ 234100, C₂H₅OH 188460

### Table 7. Some of the optimum results for different power of auxiliary power system (APS)

<table>
<thead>
<tr>
<th>Power of APS (KW)</th>
<th>Fuel</th>
<th>ANC ($/year)</th>
<th>CO2 emission (kg/year)</th>
<th>Number of panel</th>
<th>Power of electrolyser (KW)</th>
<th>LPSP</th>
</tr>
</thead>
<tbody>
<tr>
<td>275</td>
<td>CH₄</td>
<td>123326</td>
<td>121249</td>
<td>2023</td>
<td>310</td>
<td>5%</td>
</tr>
<tr>
<td></td>
<td>C₂H₆</td>
<td>170244</td>
<td>152272</td>
<td>2011</td>
<td>310</td>
<td>5%</td>
</tr>
<tr>
<td>375</td>
<td>CH₄</td>
<td>134474</td>
<td>135680</td>
<td>2015</td>
<td>310</td>
<td>1%</td>
</tr>
<tr>
<td></td>
<td>C₂H₆</td>
<td>187005</td>
<td>169482</td>
<td>2013</td>
<td>310</td>
<td>1%</td>
</tr>
<tr>
<td>500</td>
<td>CH₄</td>
<td>148068</td>
<td>147169</td>
<td>2009</td>
<td>310</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>C₂H₆</td>
<td>205056</td>
<td>183006</td>
<td>2016</td>
<td>310</td>
<td>0%</td>
</tr>
<tr>
<td>600</td>
<td>CH₄</td>
<td>160875</td>
<td>152740</td>
<td>2051</td>
<td>320</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>C₂H₆</td>
<td>219277</td>
<td>192780</td>
<td>2022</td>
<td>310</td>
<td>0%</td>
</tr>
</tbody>
</table>

#### International fuel prices

- 275: CH₄ 123326, C₂H₆ 170244, C₂H₅OH 187005
- 375: CH₄ 134474, C₂H₆ 187005, C₂H₅OH 205056
- 500: CH₄ 148068, C₂H₆ 205056, C₂H₅OH 219277
- 600: CH₄ 160875, C₂H₆ 219277, C₂H₅OH 264460

#### Iran fuel prices

- 275: CH₄ 123326, C₂H₆ 170244, C₂H₅OH 187005
- 375: CH₄ 134474, C₂H₆ 187005, C₂H₅OH 205056
- 500: CH₄ 148068, C₂H₆ 205056, C₂H₅OH 219277
- 600: CH₄ 160875, C₂H₆ 219277, C₂H₅OH 264460

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**Figure 8.** Pareto frontier of hybrid system for methane with different power of APS, Iran fuel price, MNP:3000.

**Figure 9.** Pareto frontier of hybrid system for diesel fuel with different power of APS, International fuel price, MNP:3000.
If the panel angle changes and tracks the sun, PV panel productivity increases and the number of panels decreases, so the annualized cost are reduced. In this study, the effect of seasonal and monthly panel angle change is considered. The best panel angle in Kerman (30°15'N, 56°58'E), Iran, for the most productivity in different months and seasons are brought in Tables (8,9).

These angles determined by comparing the received solar radiation for different tilt angles in every month or every season [18]. The optimum panel angle in Kerman for fixed panel angle situation is about the latitude of Kerman (30°) [32]. Figure 12. shows the effect of panel angle change on the hybrid system in international fuel price for methane. It shows that the effect of panel angle change is negligible specially when the number of panels decreases. Figure13. shows the effect of panel angle change in Iran fuel price situation for methane. Its results are the same as Figure 12. Figure 14. is plotted for diesel fuel. It is obvious that when the fuel price is high, the hybrid system goes to select more panels and so the effect of panel angle change increases, as much as, the difference between seasonal and monthly changes of panel angle are observable.
8. CONCLUSION

This hybrid system is a practical system for generating a load demand. The number of optimal solutions depends on the fuel price and MNP. If the power of the auxiliary system is high, the annualized cost and the emission of the system increase, and if it is low, the reliability of the system decreases, so the power of the auxiliary system should always be elected optimally. Methane is the best fuel for this hybrid system economically and ecologically. But if there is no gas pipeline, the best alternative fuel is propane. Seasonal or monthly change of panel angle affects the system when the fuel price is high or when the number of panels increases.

REFERENCES


